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V.Ya. Gutlyanskiĭ[1], O.V. Nesmelova[1,2], V.I. Ryazanov[1,3]

[1] Institute of Applied Mathematics and Mechanics of the NAS of Ukraine, Slov'yansk
[2] Donbas State Pedagogical University, Slov'yansk
[3] Bogdan Khmelnytsky National University of Cherkasy
E-mail: vgutlyanskii@gmail.com, star-o@ukr.net, vl.ryazanov1@gmail.com

The Dirichlet problem for the Poisson type equations in the plane

Presented by Corresponding member of the NAS of Ukraine V.Ya. Gutlyanskiĭ

We present a new approach to the study of semilinear equations of the form $\mathrm{div}[A(z)\nabla u] = f(u)$, *the diffusion term of which is the divergence uniform elliptic operator with measurable matrix functions* $A(z)$, *whereas its reaction term* $f(u)$ *is a continuous non-linear function. We establish a theorem on the existence of weak* $C(\overline{D})\cap W^{1,2}_{\mathrm{loc}}(D)$ *solutions of the Dirichlet problem with arbitrary continuous boundary data in any bounded domains D without degenerate boundary components and give applications to equations of mathematical physics in anisotropic media.*

Keywords: *Dirichlet problem, semilinear elliptic equations, quasilinear Poisson equations, anisotropic and inhomogeneous media, quasiconformal maps.*

1. Introduction. The paper is devoted to semilinear partial differential equations of the form

$$\mathrm{div}[A(z)\nabla u(z)] = f(u(z)) \tag{1}$$

in domains D of $\mathbb{C}$, where functions $f:\mathbb{R}\to\mathbb{R}$ are continuous and such that

$$\lim_{t\to\infty}\frac{f(t)}{t} = 0, \tag{2}$$

$A\in M^{2\times 2}_K(D)$, $1\leqslant K<\infty$, i.e., symmetric matrix functions $A(z)=\{a_{ij}(z)\}$, $\det A(z)=1$ with measurable entries satisfying the uniform ellipticity condition

$$\frac{1}{K}|\xi|^2 \leqslant \langle A(z)\xi,\xi\rangle \leqslant K|\xi|^2 \quad \forall\xi\in\mathbb{R}^2. \tag{3}$$

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Following paper [1], under *a weak solution* of Eq. (1), we understand a function $u\in C(\overline{D})\cap W^{1,2}_{\text{loc}}(D)$ such that, for all $\varphi\in C_0(D)\cap W^{1,2}(D)$,

$$\int\limits_D \langle A(z)\nabla u(z),\, \nabla\varphi(z)\rangle\, dm(z)+\int\limits_D f(u(z))\varphi(z)\, dm(z)=0\ . \tag{4}$$

History comments and other definitions can be found in our previous paper [2].

The paper is organized as follows. In Sections 2, one can find existence theorems for the semilinear equation (1) without boundary conditions. We study the solvability of the Dirichlet problem with arbitrary continuous boundary data for the quasilinear Poisson equations in Section 3. Section 4 is devoted to the solvability of the Dirichlet problem with continuous boundary data for the semilinear equation (1). Finally, Section 5 contains some physical applications.

2. On a weak solvability of semilinear equations. We start from the study of the solvability of the semilinear equations (1) without any boundary conditions.

Theorem 1. *Let* D *be a domain with a finite area that is not dense in* $\mathbb{C}$, $A\in M^{2\times2}_K(D)$, *and let a continuous function* $f:\mathbb{R}\to\mathbb{R}$ *satisfy condition* (2). *Then there is a weak solution* $u:D\to\mathbb{R}$ *of Eq.* (1) *which is locally Hölder-continuous in* D.

Proof. Let us extend $A(z)$ by the identity matrix I outside of D. By Theorem 4.1 in [1], if u is a weak solution of (1), then $u=U\circ\omega$, where $\omega:=\Omega|_D$ and Ω is a quasiconformal mapping of $\overline{\mathbb{C}}$ onto itself, $\Omega(\infty)=\infty$, agreed with the extended A, and U is a weak solution of the quasilinear Poisson equation

$$\Delta U(z)=h(z)\cdot f(U(z)) \tag{5}$$

with $h=J$, where J is the Jacobian of the mapping $\omega^{-1}:D_*\to D$, $D_*|:=\omega(D)$.

Note that $\overline{\mathbb{C}}\setminus D$ contains a nondegenerate (connected) component C, because D is not dense in $\mathbb{C}$, see, e.g., Corollary IV.2 and the point II.4.D in [3], see also Lemma 5.1 in [4]. Hence, $\overline{\mathbb{C}}\setminus D_*$ contains a component $C_*:=\Omega(C)$ whose boundary is a nondegenerate continuum, see again Lemma 5.1 in [4], and, by the Riemann theorem, there is a conformal mapping H of $\overline{\mathbb{C}}\setminus C_*$ onto $\mathbb{D}$.

Setting $H_*=H|_{D_*}$, we see that H_* maps D_* into $\mathbb{D}$. Moreover, the quasiconformal mapping $\omega_*:=H_*\circ\omega:D\to\mathbb{D}_*:=H_*(D_*)$ is also agreed with A in D. Thus, again by Theorem 4.1 in [1], $u=U_*\circ\omega_*$, where U_* is a weak solution of (5) with $h=J_*$ in $\mathbb{D}_*\subseteq\mathbb{D}$. Here, J_* is the Jacobian of the mapping $\omega_*^{-1}:\mathbb{D}_*\to D$.

By Remark 4.1 in [1], inversely, if U_* is a weak solution of (5) with $h=J_*$ in $\mathbb{D}_*$, then $u=U_*\circ\omega_*$ is a weak solution of (1) in D. The latter implication allows us to reduce the proof of Theorem 1 to Corollary 3 in [2] with the special $h=J_*$.

Indeed, $J_*\in L^1(\mathbb{D}_*)$, because its integral is equal to the area of the domain D, see, e.g., Theorem 3.2 in [5] and Theorem II.B.3 in [6]. Moreover, $J_*\in L^p_{\text{loc}}(\mathbb{D}_*)$ for some $p>1$, because, by the Bojarski result in [5], the first partial derivatives of the quasiconformal mapping $\omega^*:=\omega_*^{-1}:\mathbb{D}_*\to D$ are locally integrable with a power $q>2$, and $J_*=|\omega^*_w|^2-|\omega^*_{\overline{w}}|^2$, see, e.g., I.A (9) in [6].

3. Dirichlet problem for a quasilinear Poisson equation. Let D be a bounded domain in $\mathbb{C}$ *without degenerate boundary components*, i.e., any connected component of the boundary of D is not degenerated to a single point. Given a continuous boundary function $\varphi:\partial D\to\mathbb{R}$, let us denote by D_φ, the harmonic function in D that has the continuous extension to $\overline{D}$ with φ as its boundary data. Such a function exists and is unique, see, e.g., Corollary 4.1.8 and Theorem 4.2.2 in [7]. Thus, the *Dirichlet operator* D_φ is well defined in the given domains. We need not its explicit description for our goals.

By Theorem 1 in [2], we come to the following result on the existence, regularity, and representation of solutions of the Dirichlet problem for the Poisson equation in arbitrary bounded domains D in $\mathbb{C}$, where we assume that the charge density g is extended by zero outside of D.

Theorem 2. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, $\varphi:\partial D\to\mathbb{R}$ be a continuous function, and $g:D\to\mathbb{R}$ belong to the class $L^p(D)$ for $p>1$. Then the function $U:=N_g-D_{N_g^*}+D_\varphi$, $N_g^*:=N_g|_{\partial D}$, is continuous in $\overline{D}$ with $U|_{\partial D}=\varphi$, belongs to the class $W^{2,p}_{\mathrm{loc}}(D)$, and satisfies the Poisson equation $\Delta U=g$ a.e. in D. Moreover, $U\in W^{1,q}_{\mathrm{loc}}(D)$ for some $q>2$, and U is locally Hölder-continuous in D. Furthermore, $U\in C^{1,\alpha}_{\mathrm{loc}}(D)$ with $\alpha=(p-2)/p$, if $g\in L^p(D)$ for $p>2$.*

The case of quasilinear Poisson equations is reduced to the case of the linear Poisson equations again by the Leray—Schauder approach as in the last section.

Theorem 3. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, $\varphi:\partial D\to\mathbb{R}$ be a continuous function, and $h:D\to\mathbb{R}$ be a function in the class $L^p(D)$ for $p>1$. Suppose that a function $f:\mathbb{R}\to\mathbb{R}$ is continuous and satisfies* (2). *Then there is a continuous function $U:\overline{D}\to\mathbb{R}$ with $U|_{\partial D}=\varphi$ and $U|_D\in W^{2,p}_{\mathrm{loc}}(D)$ such that*

$$\Delta U(z)=h(z)\cdot f(U(z)) \quad \text{for a.e.} \quad z\in D. \tag{6}$$

Moreover, $U\in W^{1,q}_{\mathrm{loc}}(D)$ for some $q>2$ and U is locally Hölder-continuous. Furthermore, $U\in C^{1,\alpha}_{\mathrm{loc}}(D)$ with $\alpha=(p-2)/p$, if $p>2$.

Proof. If $\|h\|_p=0$ or $\|f\|_C=0$, then the Dirichlet operator D_φ gives the desired solution of the Dirichlet problem for Eq. (6), see, e.g., I.D.2 in [8]. Hence, we may assume further that $\|h\|_p\neq 0$ and $\|f\|_C\neq 0$. Set $f_*(s)=\max\limits_{|t|\leqslant s}|f(t)|$, $s\in\mathbb{R}^+$. Then the function $f_*:\mathbb{R}^+\to\mathbb{R}^+$ is continuous and nondecreasing. Moreover, $f_*(s)/s\to 0$ as $s\to\infty$ by (2).

By Theorem 1 in [9] and the maximum principle for harmonic functions, we obtain the family of operators $F(g;\tau):L^p(D)\to L^p(D)$, $\tau\in[0,1]$:

$$F(g;\tau):=\tau h\cdot f(N_g-D_{N_g^*}+D_\varphi), \quad N_g^*:=N_g|_{\partial D}, \quad \forall\tau\in[0,1]. \tag{7}$$

It satisfies all hypotheses H1-H3 of Theorem 1 in [10]. Indeed:

H1). First of all, $F(g;\tau)\in L^p(D)$ for all $\tau\in[0,1]$ and $g\in L^p(D)$, because, by Theorem 1 in [9], $f(N_g-D_{N_g^*}+D_\varphi)$ is a continuous function. Moreover,

$$\|F(g;\tau)\|_p\leqslant\|h\|_p\, f_*(2M\|g\|_p+\|\varphi\|_C)<\infty \quad \forall\tau\in[0,1].$$

Thus, by Theorem 1 in [9] and the Arzela–Ascoli theorem, see, e.g., Theorem IV.6.7 in [11], the operators $F(g;\tau)$ are completely continuous for each $\tau\in[0,1]$ and even uniformly continuous with respect to the parameter $\tau\in[0,1]$.

H2). The index of the operator $F(g;0)$ is obviously equal to 1.

H3). By Theorem 1 in [9] and the maximum principle for harmonic functions, we have the estimate for solutions $g \in L^p$ of the equations $g = F(g; \tau)$:

$$\| g \|_p \leqslant \| h \|_p \, f_*(2M \| g \|_p + \| \varphi \|_C) \leqslant \| h \|_p \, f_*(3M \| g \|_p)$$

whenever $\| g \|_p \geqslant \| \varphi \|_C / M$, i.e. then it should be

$$\frac{f_*(3M \| g \|_p)}{3M \| g \|_p} \geqslant \frac{1}{3M \| h \|_p}, \tag{8}$$

and, hence, $\| g \|_p$ should be bounded in view of condition (2).

Thus, by Theorem 1 in [10], there is a function $g \in L^p(D)$ such that $g = F(g; 1)$ and, consequently, by Theorem 1 in [2], the function $U := N_g - D_{N_g^*} + D_\varphi$ gives the desired solution of the Dirichlet problem for Eq. (6).

4. Dirichlet problem with continuous data for semilinear equations. By the factorization theorem from [1], the study of the semilinear equations (1) in bounded domains without degenerate boundary components D is reduced, by means of a suitable quasiconformal change of variables, to the study of the corresponding quasilinear Poisson equations (6).

Theorem 4. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, $A \in M_K^{2\times 2}(D)$, $\varphi : \partial D \to \mathbb{R}$ be an arbitrary continuous function, and $f : \mathbb{R} \to \mathbb{R}$ be a continuous function satisfying condition* (2).

Then there is a weak solution $u : D \to \mathbb{R}$ from the class $C(D) \cap W_{\mathrm{loc}}^{1,2}(D)$ of Eq. (1) *which is locally Hölder-continuous in D and continuous in $\overline{D}$ with $u|_{\partial D} = \varphi$.*

Proof. Let us extend, by definition, $A \equiv I$ outside of D. By Theorem 4.1 in [1], if u is a weak solution of the equation, then $u = U \circ \omega$, where $\omega := \Omega|_D$, and Ω is a quasiconformal mapping of $\mathbb{C}$ onto itself agreed with the extended A, and U is a weak solution of Eq. (6) with $h = J$, where J is the restriction of the Jacobian of the mapping $\Omega^{-1} : \mathbb{C} \to \mathbb{C}$ to the domain $D_* := \Omega(D)$.

Inversely, by Remark 4.1 in [1], we see that if U is a weak solution of (6) with $h = J$, then $u = U \circ \omega$ is a weak solution of our equation. The latter allows us to reduce Theorem 4 to Theorem 3. Indeed, $\overline{D}_* := \Omega(\overline{D})$ is compact. By the Bojarski result in [5], the generalized derivatives of the quasiconformal mapping $\Omega^* := \Omega^{-1} : \mathbb{C} \to \mathbb{C}$ are locally integrable with some power $q > 2$. Note also that the Jacobian J of its restriction $\omega^* := \Omega^*|_{D_*}$ is equal to $|\omega_w^*|^2 - |\omega_{\overline{w}}^*|^2$, see, e.g., I.A(9) in [6]. Consequently, $J \in L^p(D_*)$ for some $p > 1$.

5. On some applications to physical problems. Theorems 3 and 4 can be applied to some physical problems. The first circle of such applications is relevant to reaction-diffusion problems. Problems of this type are discussed in [12], p. 4, and, in detail, in [13]. A nonlinear system is obtained for the density u and the temperature T of the reactant. By eliminating T, the system can be reduced to the equation

$$\Delta u = \lambda \cdot f(u) \tag{9}$$

with $h(z) \equiv \lambda > 0$ and, for isothermal reactions, $f(u) = u^q$, where $q > 0$ is called the order of the reaction. It turns out that the density of the reactant u may be zero in a subdomain called a *dead core*. A particularization of results in Chapter 1 of [12] shows that a dead core may exist just if and only if $0<q<1$ and λ is large enough, see also the corresponding examples in [1]. In this connection, the following statements may be of independent interest.

Corollary 1. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, $\varphi:\partial D\to\mathbb{R}$ be a continuous function, and $h:D\to\mathbb{R}$ be a function in the class $L^p(D)$, $p>1$. Then there exists a continuous function $u:\overline{D}\to\mathbb{R}$ with $u|_{\partial D}=\varphi$ such that $u\in W^{2,p}_{\text{loc}}(D)$ and*

$$\Delta u(z)=h(z)\cdot u^q(z),\quad 0<q<1 \tag{10}$$

a.e. in D. Moreover, $u\in W^{1,\beta}_{\text{loc}}(D)$ for some $\beta>2$, and u is locally Hölder-continuous in D. Furthermore, $u\in C^{1,\alpha}_{\text{loc}}(D)$ with $\alpha=(p-2)/p$, if $p>2$.

Corollary 2. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, and $\varphi:\partial D\to\mathbb{R}$ be a continuous function. Then there is a continuous function $u:\overline{D}\to\mathbb{R}$ with $u|_{\partial D}=\varphi$ such that $U\in C^{1,\alpha}_{\text{loc}}(D)$ for all $\alpha\in(0,1)$, $u\in W^{2,p}_{\text{loc}}(D)$ for all $p\in[1,\infty)$ and*

$$\Delta u(z)=u^q(z),\quad 0<q<1,\quad \text{a.e. in } D. \tag{11}$$

Note also that certain mathematical models of a thermal evolution of a heated plasma lead to nonlinear equations of the type (9). Indeed, it is known that some of them have the form $\Delta\psi(u)=f(u)$ with $\psi'(0)=\infty$ and $\psi'(u)>0$, if $u\neq 0$ as, for instance, $\psi(u)=|u|^{q-1}u$ under $0<q<1$, see e.g. [12]. With the replacement of the function $U=\psi(u)=|u|^q\cdot\operatorname{sign}u$, we have that $u=|U|^Q\cdot\operatorname{sign}U$, $Q=1/q$, and, with the choice $f(u)=|u|^{q^2}\cdot\operatorname{sign}u$, we come to the equation $\Delta U=|U|^q\cdot\operatorname{sign}U=\psi(U)$.

Corollary 3. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, and $\varphi:\partial D\to\mathbb{R}$ be a continuous function. Then there is a continuous function $U:\overline{D}\to\mathbb{R}$ with $U|_{\partial D}=\varphi$ such that $U\in C^{1,\alpha}_{\text{loc}}(D)$ for all $\alpha\in(0,1)$, $U\in W^{2,p}_{\text{loc}}(D)$ for all $p\in[1,\infty)$ and*

$$\Delta U(z)=|U(z)|^{q-1}\,U(z),\quad 0<q<1,\quad \text{a.e. in } D. \tag{12}$$

Moreover, recall that, in the combustion theory, see, e.g., [14], [15] and the references therein, the following model equation

$$\frac{\partial u(z,t)}{\partial t}=\frac{1}{\delta}\cdot\Delta u+e^u,\quad t\geqslant 0,\quad z\in D, \tag{13}$$

takes a special place. Here, $u\geqslant 0$ is the temperature of the medium, and δ is a certain positive parameter. We restrict ourselves here by the stationary case, although our approach makes it possible to study the parabolic equation (13), see [1]. Namely, Eq. (6) appears here with $h\equiv\delta>0$ and the function $f(u)=e^{-u}$ that is bounded.

Corollary 4. *Let D be a bounded domain in $\mathbb{C}$ without degenerate boundary components, and $\varphi:\partial D\to\mathbb{R}$ be a continuous function. Then there is a continuous function $U:\overline{D}\to\mathbb{R}$ with $U|_{\partial D}=\varphi$ such that $U\in C^{1,\alpha}_{\text{loc}}(D)$ for all $\alpha\in(0,1)$, $U\in W^{2,p}_{\text{loc}}(D)$ for all $p\in[1,\infty)$ and*

$$\Delta U(z)=\delta\cdot e^{U(z)},\quad \text{a.e. in}\quad D. \tag{14}$$

Specifying the reaction term $f(u)$ of the semilinear equation (1), we also arrive, by Theorem 4, at the following statements concerning some specific problems of mathematical physics in inhomogeneous and anisotropic media.

Corollary 5. *Let* D *be a bounded domain in* $\mathbb{C}$ *without degenerate boundary components,* $A\in M_K^{2\times2}(D)$, *and* $\varphi:\partial D\to\mathbb{R}$ *be a continuous function. Then there is a continuous function* $u:\overline{D}\to\mathbb{R}$ *with* $u|_{\partial D}=\varphi$ *which is locally Hölder-continuous in* D, *and it is a weak solution in* D *for the equation*

$$\operatorname{div}[A(z)\nabla u(z)]=u^q(z),\quad 0<q<1. \tag{15}$$

Corollary 6. *Let* D *be a bounded domain in* $\mathbb{C}$ *without degenerate boundary components,* $A\in M_K^{2\times2}(D)$, *and* $\varphi:\partial D\to\mathbb{R}$ *be a continuous function. Then there is a continuous function* $u:\overline{D}\to\mathbb{R}$ *with* $u|_{\partial D}=\varphi$ *which is locally Hölder-continuous in* D, *and it is a weak solution in* D *for the equation*

$$\operatorname{div}[A(z)\nabla u(z)]=|u(z)|^{q-1}u(z),\quad 0<q<1. \tag{16}$$

Corollary 7. *Let* D *be a bounded domain in* $\mathbb{C}$ *without degenerate boundary components,* $A\in M_K^{2\times2}(D)$, *and* $\varphi:\partial D\to\mathbb{R}$ *be a continuous function. Then there is a continuous function* $u:\overline{D}\to\mathbb{R}$ *with* $u|_{\partial D}=\varphi$ *which is locally Hölder continuous in* D, *and it is a weak solution in* D *for the equation*

$$\operatorname{div}[A(z)\nabla u(z)]=e^{\alpha u(z)},\quad \alpha\in\mathbb{R}. \tag{17}$$

Finally, we note that the statements given above remain to hold, if the reaction terms in Eqs. (15)-(17) are multiplied by arbitrary functions $C\in L^\infty(D)$.

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В.Я. Гутлянський [1], *О.В. Нєсмєлова* [1, 2], *В.І. Рязанов* [1, 3]

[1] Інститут прикладної математики і механіки НАН України, Слов'янськ
[2] Донбаський державний педагогічний університет, Слов'янськ
[3] Черкаський національний університет ім. Богдана Хмельницького
E-mail: vgutlyanskii@gmail.com, star-o@ukr.net, vl.ryazanov1@gmail.com

ЗАДАЧА ДІРІХЛЕ ДЛЯ РІВНЯНЬ ТИПУ ПУАССОНА НА ПЛОЩИНІ

Запропоновано новий підхід до вивчення напівлінійних рівнянь виду $\operatorname{div}[A(z)\nabla u] = f(u)$, дифузний член яких є дивергентним рівномірно еліптичним оператором з вимірними матричними функціями $A(z)$, тоді як його реакційний член $f(u)$ є неперервною нелінійної функцією. Доведено теорему про існування слабких $C(\overline{D}) \cap W^{1,2}_{\mathrm{loc}}(D)$ розв'язків задачі Діріхле з довільними неперервними граничними даними в довільних обмежених областях D без вироджених граничних компонент і дано застосування до рівнянь математичної фізики в анізотропних середовищах.

Ключові слова: *задача Діріхле, напівлінійні еліптичні рівняння, квазілінійне рівняння Пуассона, анізотропні і неоднорідні середовища, квазіконформні відображення.*

В.Я. Гутлянский [1], *О.В. Несмелова* [1, 2], *В.И. Рязанов* [1, 3]

[1] Институт прикладной математики и механики НАН Украины, Славянск
[2] Донбасский государственный педагогический университет, Славянск
[3] Черкасский национальный университет им. Богдана Хмельницкого
E-mail: vgutlyanskii@gmail.com, star-o@ukr.net, vl.ryazanov1@gmail.com

ЗАДАЧА ДИРИХЛЕ ДЛЯ УРАВНЕНИЙ ТИПА ПУАССОНА НА ПЛОСКОСТИ

Предложен новый подход к изучению полулинейных уравнений вида $\operatorname{div}[A(z)\nabla u] = f(u)$, диффузионный член которых является дивергентным равномерно эллиптическим оператором с измеримыми матричными функциями $A(z)$, тогда как его реакционный член $f(u)$ является непрерывной нелинейной функцией. Доказана теорема о существовании слабых $C(\overline{D}) \cap W^{1,2}_{\mathrm{loc}}(D)$ решений задачи Дирихле с произвольными непрерывными граничными данными в любых ограниченных областях D без вырожденных граничных компонент и даны приложения к уравнениям математической физики в анизотропных средах.

Ключевые слова: *задача Дирихле, полулинейные эллиптические уравнения, квазилинейные уравнения Пуассона, анизотропные и неоднородные среды, квазиконформные отображения.*